The variable resistor was set at every data point so that the current would be in 0.05A intervals. The terminal voltage was measured at each current. Uncertainties were obtained by the fluctuations in the displayed voltage and current—the maximum fluctuation of the current and voltage, quite conveniently, was ± 0.01A and ± 0.01V.

Results Table

<Table 1> Terminal Voltage and Current through circuit

Current through circuit / A	Terminal Voltage / V
± 0.01	± 0.01
0.80	0.84
0.75	1.06
0.70	1.42
0.65	1.61
0.60	1.90
0.55	2.16
0.50	2.39
0.45	2.67
0.40	2.97
0.35	3.17
0.30	3.44
0.25	3.70
0.20	3.92
0.15	4.22

Graph

<Graph 1> Thermal Voltage against Current through circuit (attached)

Derivation of the e.m.f. of the cell

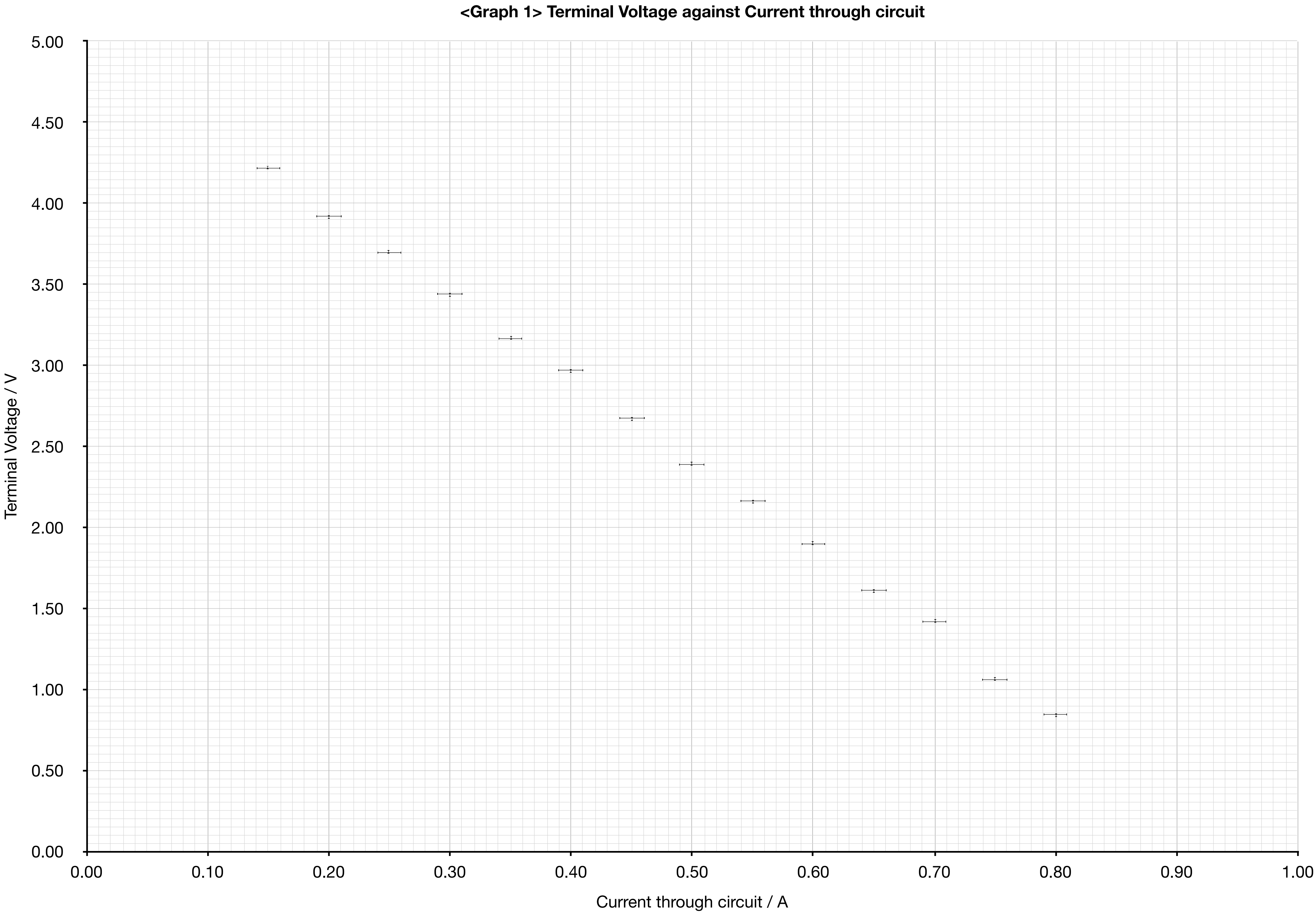
The virtual DC power supply was set to 5.00 ± 0.01 V, equivalent to the e.m.f of the virtual power supply, which was controlled to be constant. The uncertainty includes the slight fluctuations in voltage throughout the experiment.

The e.m.f. of a cell is defined as its terminal voltage when no current is flowing. Although this is impossible to measure, we can reasonably extrapolate and obtain this value by measuring the terminal voltage along different currents through the cell.

As the data points showed a linear relationship, we can extrapolate the best-fit line to obtain a value of the terminal voltage when there is no current, which is indicated as the y-intercept in the graph. Using the extrapolation of the worst-fit lines, we can determine the y-intercept, and therefore the e.m.f. of the cell:

$4.92\ V \leq \text{(e.m.f)} \leq 5.07\ V$ (Shown in graph)

$\text{(e.m.f)} = 5.00 \pm 0.08\ V$



<Graph 1> Terminal Voltage against Current through circuit
Terminal Voltage / V
Current through circuit / A
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

This result fits our expectation, as we have fixed the e.m.f. of our virtual cell to 5.00 ± 0.01 V, which is inside the range obtained from data.

Derivation of the internal resistance of the cell

The virtual internal resistance was simulated by two 10 ± 5% Ω resistors connected in parallel. Therefore, the simulated internal resistance is expected to be 5 ± 2.5% Ω, or 5 ± 0.13 Ω

We can also use simple calculus to determine the internal resistance of the cell. If we take the internal resistance as R_I and the variable resistor's resistance as R_V, we can see:

	Voltage / V	Current / A	Resistance / Ω
Whole Circuit	5	$\frac{5}{R_I + R_V}$	$R_I + R_V$
At internal resistance	$\frac{5R_I}{R_I + R_V}$	$\frac{5}{R_I + R_V}$	R_I
At variable resistance	$\frac{5R_V}{R_I + R_V}$	$\frac{5}{R_I + R_V}$	R_V

if we assume that the voltage at the variable resistor (i.e. terminal voltage) as a function of R_V, $V(R_V)$, and the current at the variable resistor as a function of R_V, $I(R_V)$, we can see that the x-axis of the graph is $I(R_V)$ and the y-axis, $V(R_V)$.

$$V(R_V) = \frac{5R_V}{R_I + R_V}$$

$$I(R_V) = \frac{5}{R_I + R_V}$$

Let $k(I(R_V))$ be the function of the graph, i.e. that maps the current at the variable resistor to the terminal voltage. Then, the following relationship, by definition, holds true:

$$k \circ I(R_V) = V(R_V)$$

If the find the derivative of function $V(R_V)$, using the derivative rules,

$$V'(R_V) = k'(I(R_V)) \cdot I'(R_V)$$

$$k'(I(R_V)) = \frac{V'(R_V)}{I'(R_V)}$$

Using the first principle, we can calculate the derivative of the function of the graph, which will result in the gradient represented as a certain equation with the resistances as the variables:

$$k'(I(R_V)) = \frac{V'(R_V)}{I'(R_V)} = \lim_{h \to 0} \frac{\frac{V(R_V + h) - V(R_V)}{h}}{\frac{I(R_V + h) - V(R_V)}{h}}$$

$$= \lim_{h \to 0} \frac{V(R_V + h) - V(R_V)}{I(R_V + h) - V(R_V)}$$

$$= \lim_{h \to 0} \frac{\frac{5(R_V + h)}{R_I + (R_V + h)} - \frac{5R_V}{R_I + R_V}}{\frac{5}{R_I + (R_V + h)} - \frac{5}{R_I + R_V}}$$

$$= \lim_{h \to 0} \frac{(R_V + h) \cdot (R_I + R_V) - R_V \cdot (R_I + R_V + h)}{(R_I + R_V) - (R_I + R_V + h)}$$

$$= \lim_{h \to 0} \frac{R_V \cdot R_I + R_V^2 + R_I \cdot h + R_V \cdot h - (R_V \cdot R_I + R_V^2 + R_V \cdot h)}{-h}$$

$$= \lim_{h \to 0} \frac{R_I \cdot h}{-h}$$

$$= \lim_{h \to 0} (-R_I)$$

$$= -R_I$$

$$k'(I(R_V)) = -R_I$$

$$\therefore |k'(I(R_V))| = R_I$$

Therefore, in theory, the absolute value of the gradient of the best-fit line of the function is expected to be the the internal resistance.

$$k'(I(R_V)) = (\text{Gradient})$$

$$|(\text{Gradient})| = R_I$$

Using our worst-fit lines, we can calculate the range of the gradient of the function of the graph:

$$\frac{-3.40}{0.69}\ V \cdot I^{-1} \leq (\text{Gradient}) \leq \frac{-2.80}{0.54}\ V \cdot I^{-1}$$

$$4.93\ V \cdot I^{-1} \leq |(\text{Gradient})| \leq 5.19\ V \cdot I^{-1}$$

$$|(\text{Gradient})| = 5.06 \pm 0.13\ V \cdot I^{-1}$$

Therefore, we can determine the internal resistance, R_I:

$$R_I = 5.06 \pm 0.13\ V \cdot I^{-1}$$

This range of uncertainty coincides with the value of the virtual internal resistance of 5 ± 0.13 Ω, and therefore fits our expectation.

Evaluation

It is assumed that the experiment was conducted without the knowledge of the true internal resistance and the e.m.f of the cell, and therefore that they were invisible to the measurer.

The biggest relative uncertainty was in the measurement of the current through the circuit, with a relative uncertainty ranging from 1.25% to 6.67%, while the measurement of the terminal voltage had a lower relative uncertainty ranging from 0.24% to 1.19%. Due to these uncertainties, the extrapolation of the y-intercept of the graph, which signifies the e.m.f, had an uncertainty of 1.6%, within the range of the drawn worst-fit lines. Also, calculation of the gradient in the process of calculating internal resistance yielded an uncertainty of 2.57%, a fairly significant value.

A consistent problem that prevented a simple solution to reducing the uncertainties was the issue of voltage and current fluctuation. The ammeter and voltmeter's readings fluctuated within ±0.01A, which were recored as the uncertainties of the measurements. Therefore, a more precise current/voltage measurement cannot, in this case, yield a lower uncertainty.

Another potential solution would be to increase the values of the current and voltage to reduce the relative uncertainty, possibly using a variable resistor with higher maximum resistance. However, this is not effective in increasing the values of both the voltage and the current, as the two values are measured to be inversely correlated, and therefore is impossible to increase at the same time.

Therefore, the best feasible solution would be to include a smoothing capacitor in various locations in the circuit (most importantly, at the wire connected to the power supply) to remove the small fluctuations in the voltage. Assuming the smoothing capacitor is of high quality and good enough to smooth the voltage and current fluctuations to lower than the sensitivity of the ammeter and voltmeter(detailed following on), it would be possible to (i) use the inherent uncertainty of the voltmeter, ±0.005V, and (ii) alter the connection to the ammeter to a more sensitive one that distinguishes between single miliAmperes, to reduce the uncertainty of the current reading to a smaller, inherent uncertainty value of the ammeter of ±0.0005A. This would theoretically reduce the maximum relative uncertainty of the terminal voltage to 0.6%, and the current to 0.3%

It would also be necessary, if the previous suggests were implemented, to use a bigger pice of paper to more accurately show the uncertainty bars on the graph, as in the current graph, the graticules were too big relative to the uncertainties. Using a bigger piece of paper with finer graticules, or calculating the gradient and the y-intercept using digital methods, would be required to reduce the uncertainty of the final extrapolated and calculated values of e.m.f. and internal resistance, after above solutions are implemented.