I. Expectations

In this experiment, we will measure the spring constant in two separate methods, and judge whether the two values agree. **The first method** will reference Hooke's Law to measure the extension of the spring depending on the acting force, according to the following relationship:

$$F = kx$$

Where F is the force acting on the spring, x is the extension of the spring, and k is the spring constant. The graph will be plotted as the *weight (force) on the spring*, against the *extension of the spring*.

The second method will use measure period of oscillation of an attached mass (as Hooke's law will guarantee that the force—thus the acceleration—acting on the mass is proportional and opposite to the displacement from equilibrium) as the motion of the mass is a *simple harmonic motion*. The following relationship will be used:

$$\omega = \sqrt{\frac{k}{m}}$$

Where ω is the angular frequency of oscillation, m is the mass of the weight, and k is the spring constant. ω is obtained indirectly by using the relationship $\omega = \frac{2\pi}{T}$, where T is the period of oscillation. The relationship then is substituted and rearranged to:

$$\frac{4\pi^2}{T^2} = k \cdot \frac{1}{m}$$

We will, therefore, plot the graph of the *square of angular frequency*, against the *reciprocal of the mass of weights*, to surface a linear relationship and obtain the spring constant.

II. Methodology

Method 1: Hooke's Law

Procedure:
1. Measure the mass of the weights and determine their uncertainties.
2. Set up apparatus according to Diagram 1 in Section III.
3. Note spring length without any attached mass
4. Attach mass of spring, and measure the extension of the spring.
5. Repeat step 4, varying the total mass of the weights.

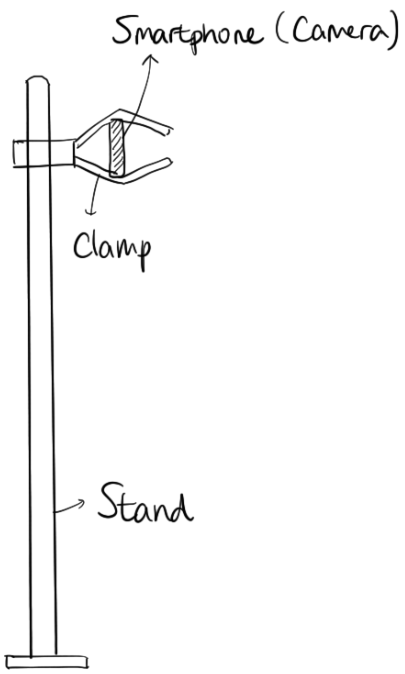
Method 2: Simple Harmonic Motion

Procedure:
1. Measure the mass of the weights and determine their uncertainties.
2. Set up apparatus according to Diagram 2 in Section III.
3. Attach mass of spring, and extend and release adequately to ensure minimal unintended deviation from desired motion
4. Measure the period of oscillation of weight, by analyzing a slow-motion recording of the simple harmonic motion
5. Repeat step 5, varying the total mass of the weights.

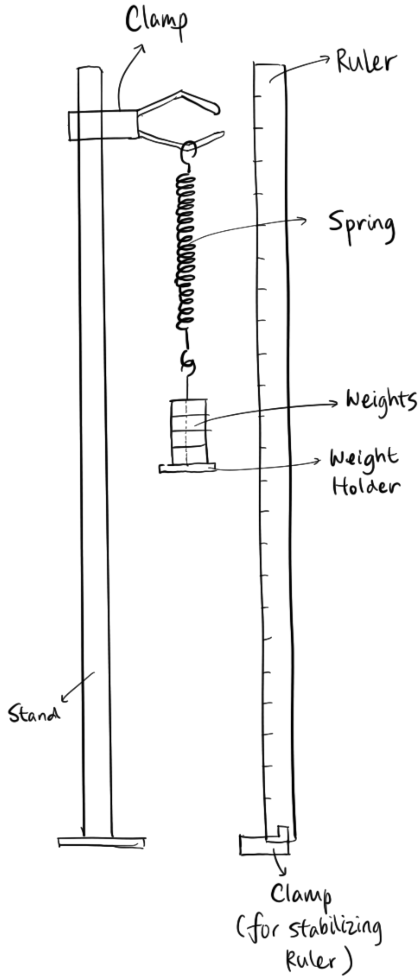
Notable Apparatus:
- Digital Balance
- Hooked Weight Holder
- Slotted Weights*
- Spring
- Ruler

* It is crucial to distinguish between the apparatus, the metal *weights*, and the physical value, *weight*, measured in Newtons.

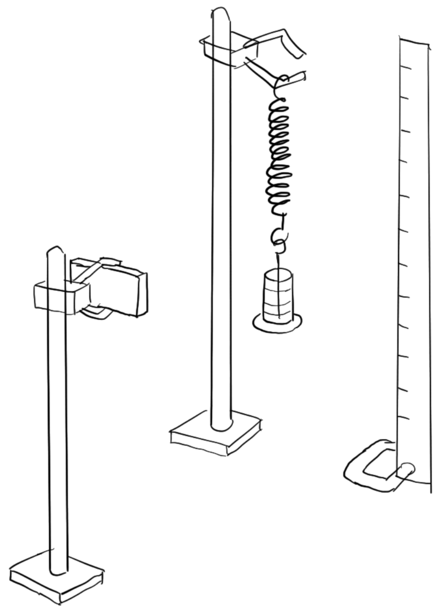
III. Diagrams



[Diagram 1] Smartphone (slow motion camera) attached to Stand and Clamp (Side View)



[Diagram 2] Spring and Ruler attached to Stand and Clamp (Front View)



[Drawing 3] Complete Apparatus (Diagonal View)

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IV. Table and Data Processing

The mass of the weights were increased in 50g intervals. All weights were chosen to have ±1% uncertainty or less each, and therefore will have less than ±1% uncertainty combined as well. These measurements, as well as the value of physical constants, are listed in *Table 3*.

For the first method, the weight (force) on the spring is calculated using $w = mg$, where g is the gravitational field strength, taken as $9.81m \cdot s^{-2}$. For example, the force that a 150g metal weight imposes on the spring is calculated as:$(150g \pm 1\%) \cdot 10^{-3} \cdot 9.81 = (1.472 \pm 1\%) = 1.472 \pm 0.015\ N$.

The extension of the spring is measured with a ruler with graticules of 1mm, and taken down by using a straight stick to identify the spring's accurate extended length. Due to the minor inaccuracies and slight rocking of the spring, the uncertainty is taken to be ±1mm or 0.001m

For the second method, the period measurement is taken among five oscillations, the length of which is calculated by counting the frames between the start and end of the five oscillation of the slow-motion recording, taken at 240 frames per second. Therefore, the period of one oscillation, given a certain mass on the spring is:

$$T = \frac{(\textbf{end frame number} - \textbf{start frame number})}{240 \cdot 5}$$

The uncertainty is taken as $\pm \frac{1}{120 \cdot 5}\ s \approx \pm\ 0.002\ s$, taking into account the shutter lag of the digital camera sensor and minor inaccuracies in frame counting. This uncertainty is propagated as the square of the angular frequency is calculated using the above mentioned relationship $\omega = \frac{2\pi}{T}$:

$$\omega^2 = \frac{4\pi^2}{T^2}$$

For example, the square of the angular frequency of the oscillation when the attached mass is 150g, is calculated as:

$$T_{150g} = \frac{5248 - 4608}{240 \cdot 5} = 0.533 \pm 0.002\ s$$

$$\omega^2_{150g} = \frac{4\pi^2}{(0.533 \pm 0.002)^2} = \frac{4\pi^2}{0.284089} \pm 0.7\,\% = 139 \pm 0.7\,\% = 139 \pm 1\ Hz^2$$

The reciprocal of mass of weights are simply calculated as $\frac{1}{m}$, with its relative uncertainty unchanged. For example, for a weight of mass 150g, $\frac{1}{m} = \frac{1}{150 \cdot 10^{-3} \pm 1\,\%} = 6.67 \pm 1\,\%$.
The significant figures are chosen according to the absolute uncertainty of each value,
e.g. $6.67 \cdot 1\,\% = 0.0667,\ \therefore\ 6.67 \pm 0.07$. **or** 6.67 ±1%

[Table 3] Measured Constants

Mass of weights / g	Mass of weight holder / g	Gravitational Field Strength / $m \cdot s^{-2}$	Pi (π)
50 ± 0.5 (1%)	100 ± 1 (1%)	9.81	3.1416

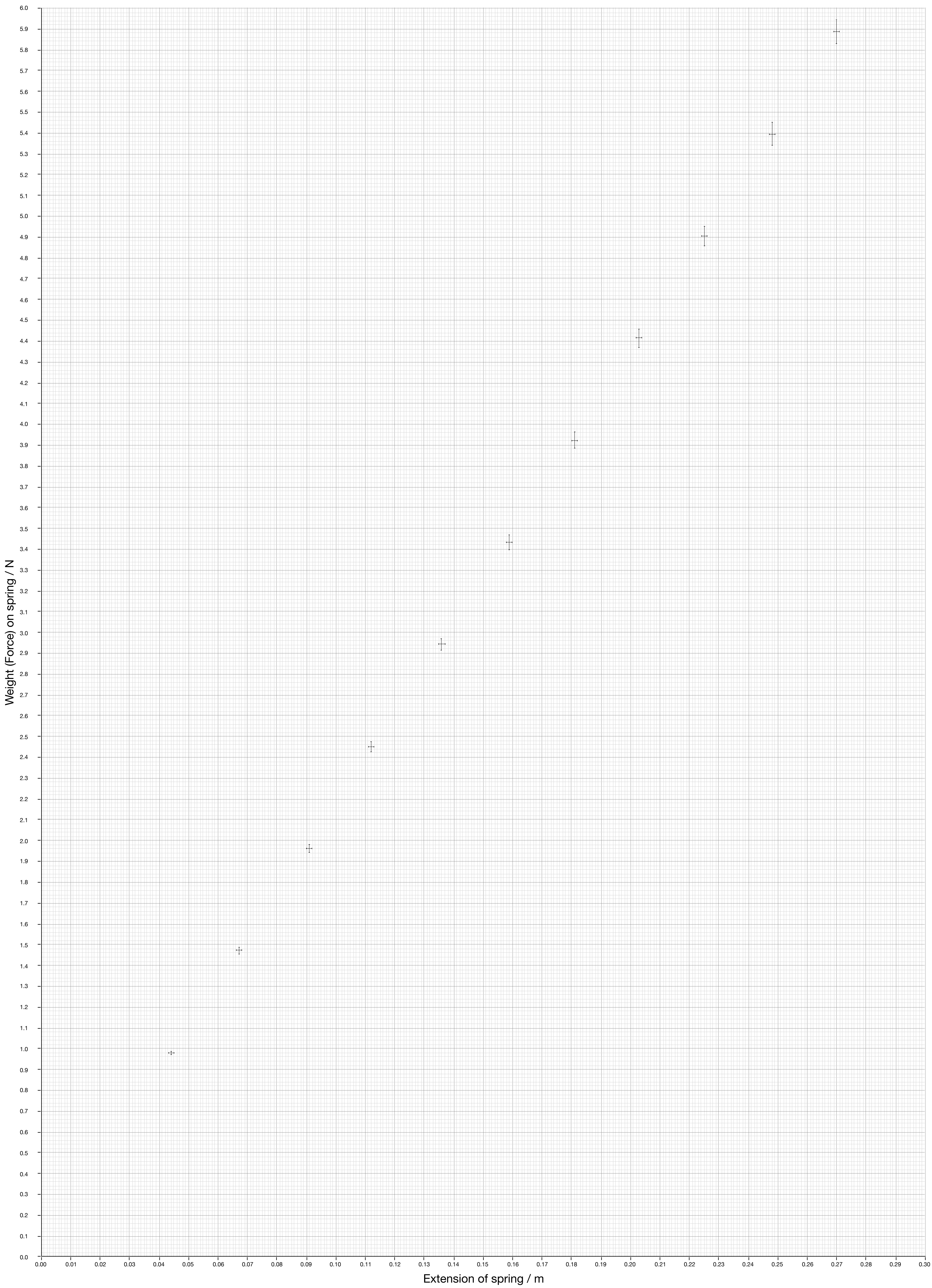
[Method 1: Table 1] Extension of Spring due to force by weights of different masses

Total mass of weights on spring (m) / g	**Weight (Force) on spring (F) / N**	**Extension of spring (x) / m**
± 1%	± 1%	**± 0.001**
100	0.98	0.044
150	1.47	0.067
200	1.96	0.091
250	2.45	0.112
300	2.94	0.136
350	3.43	0.159
400	3.92	0.181
450	4.41	0.203
500	4.91	0.225
550	5.40	0.248
600	5.89	0.270

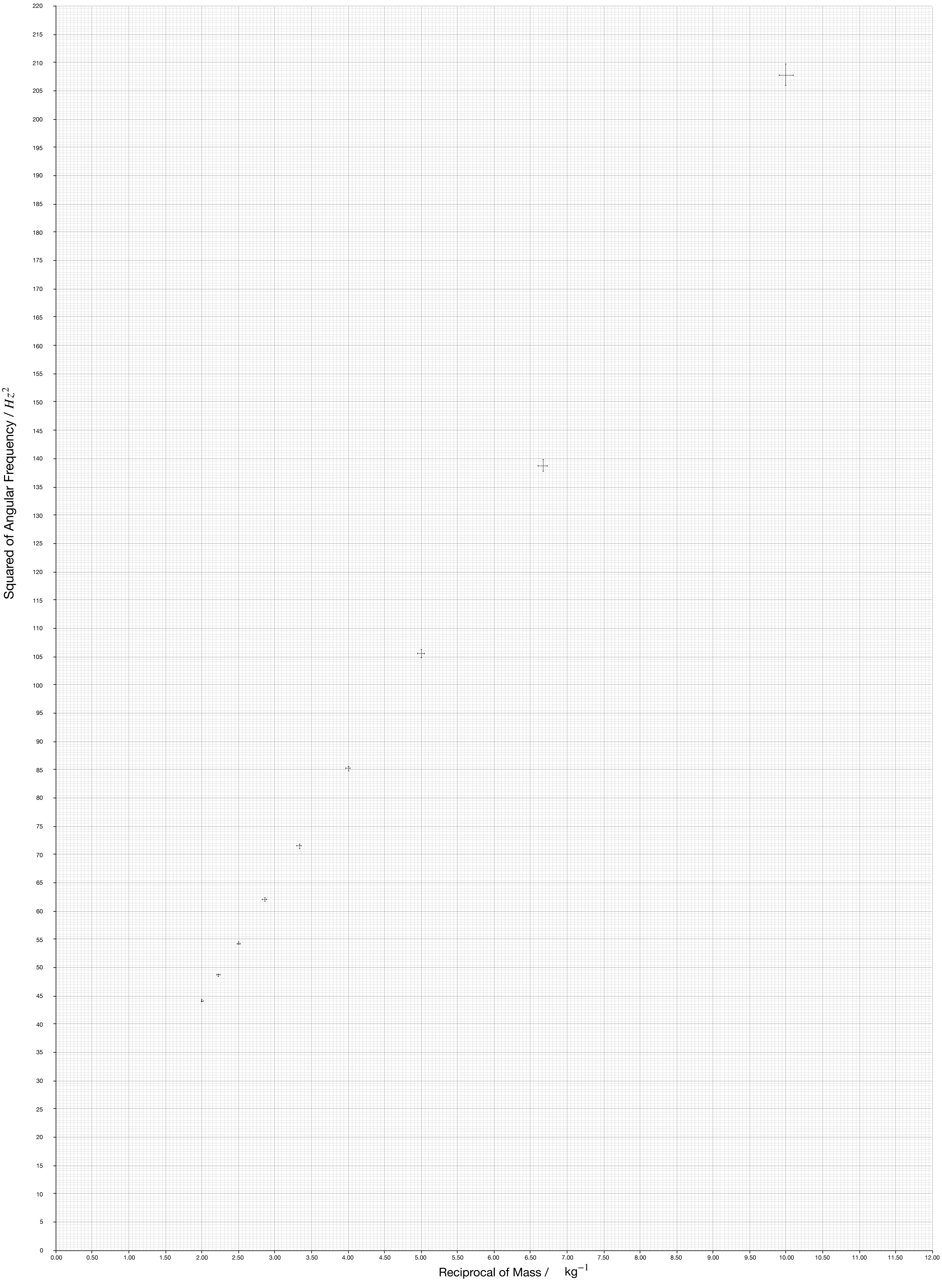
[Method 2: Table 2] Period of oscillation and Squared Angular Frequency by weights of different masses

Raw Data	*Processed Data*	*Raw Data*		*Processed Data*	
Mass of weights attached to spring (m) / g	**Reciprocal of mass of weights ($\frac{1}{m}$) / kg^{-1}**	**start frame**	**end frame**	**Period (T) / s**	**Square of Angular Frequency (ω) / Hz^2**
± 1%	**± 1%**	**–**	**–**	**± 0.002**	**–**
100	10.00	3771	4294	0.436	208 ± 2
150	6.67	4608	5248	0.533	139 ± 1
200	5.00	1279	2013	0.612	105.5 ± 0.7
250	4.00	751	1568	0.681	85.1 ± 0.5
300	3.33	697	1589	0.743	71.4 ± 0.4
350	2.86	661	1619	0.798	61.9 ± 0.3
400	2.50	839	1862	0.853	54.3 ± 0.3
450	2.22	909	1990	0.901	48.6 ± 0.2
500	2.00	1285	2420	0.946	44.1 ± 0.2

[Method 1: Graph 1] Weight (Force) on Spring Against Extension of Spring



[Method 2: Graph 2] Angular Frequency of Oscillation against Reciprocal of Attached Mass



V. Graph Processing

Attached: [Method 1: Graph 1] Weight (Force) on Spring Against Extension of Spring

As the graph is plotted as the weight (force) on the spring, against the extension, using Hook's law we can see that the gradient of the graph indicates the spring constant.

$$\frac{F}{x} = k_1$$

The spring constant $k_1$and its uncertainty, obtained from this first method, is calculated from the graph to be the following:

$$\frac{4.48}{0.212} \leq k_1 \leq \frac{4.85}{0.226}$$

$$21.13 \leq k_1 \leq 22.46$$

$$k_1 = 21.8 \pm 0.7 N \cdot m^{-1}$$

Attached: [Method 2: Graph 2] Angular Frequency of Oscillation against Reciprocal of Attached Mass

The gradient of the graph, also in this case, is equal to the spring constant, as the graph was plotted with the squared of angular frequency against the reciprocal of mass.

$$\frac{4\pi^2}{T^2} = k_2 \cdot \frac{1}{m}$$

$$\omega^2 = k_2 \cdot \frac{1}{m}$$

The spring constant k_2 and its uncertainty, obtained from this second method, is calculated from the graph to be the following:

$$\frac{162.5}{8.15} \leq k_2 \leq \frac{96}{4.6}$$

$$19.94 \leq k_2 \leq 20.87$$

$$k_2 = 20.4 \pm 0.5 N \cdot m^{-1}$$

VI. Conclusion

The range of the two resultant spring constants, which is predicted to agree, k_1 and k_2, does not overlap, despite being very close to each other.

$$k_1 = 21.8 \pm 0.7 N \cdot m^{-1}$$
$$k_2 = 20.4 \pm 0.5 N \cdot m^{-1}$$

This does not meet our theoretical expectations that the two values will overlap. The likely explanation is that there were systematical errors in Method 1, Method 2, or both, which caused an overall deviation between the two values. These errors will be noted in the following section.

VII. Evaluation

The most significant error is the disagreement between the values of the spring constant obtained from Method 1 and Method 2. We can suspect that the error is systematic, as in Graph 2, the vertical intercept of the worst-fit lines do not enclose the origin (which it should, as $\frac{4\pi^2}{T^2}$ and $\frac{1}{m}$ has a directly proportional relationship without other terms to shift the vertical intercept). As the meaning of the axis in Graph 2 are difficult to evaluate, though, the cause of the problem is difficult to identify. We can think of some systematic errors that may have caused this problem, as well as their respective solutions:

(i) A slight change in the location of the ruler / height of clamp holding the spring / bending of hood

The most likely explanation is that during the experiment there was a slight movement in the location of the ruler relative to the spring, by a slight bump or an unstable table, or a change in the bend of the hook holding the spring. This can be resolved by being more careful to keeping the equipment consistent, and re-measuring default values at regular intervals.

Note, however, that in Method 2, dampening (decrease in amplitude due to energy loss from system) would not have significantly altered the measurements, as the frequency is not correlated with amplitude.

(ii) Change in spring constant due to repetitive, excessive strain

It is possible, although unlikely, that the spring was inconsistent in its behavior, due to the large amount of load imposed on it throughout the experiment. This can be resolved by using a spring capable of higher loads, or by reducing the force imposed on the spring (though at a cost of reduced data reliability).

(iii) Shutter lag in recording of oscillations, misidentification of start and end frames

Every digital video camera requires processing each frame and saving it to a storage device. In some cases, especially in slow motion videos, there may be problems with saving the exact frame at the exact moment in time; delays in processing and storing the data may result in slight deviations in what is shown in the frame, and what actually happened at that moment. Alternatively, digital video compression can result in frames that are indistinguishable from neighboring frames, or a slight misrepresentation of the real location of each object.

This can be resolved simply by using photogates (which the teacher recommended, but which I declined due to the inconvenient setup), which have a much simpler mechanism, is designed for experimental use, and is free from the problems mentioned above. A lesson to be learnt is to follow the recommendations of one's teacher when choosing equipment.

Regarding the uncertainty of the measured values (ignoring possible systematic errors) the uncertainty of the period was the smallest, ranging from ±0.46% to ±0.21%. Other values, such as mass of weights (±1%), and the extension of the spring (± 0.37% to ± 2.3%) also had fairly small uncertainties.

The biggest cause of uncertainty, the measurement of the extension of the spring, can be reduced by using infra-red distance sensors facing upwards from the floor, to the spring, that would measure the distance from it to the weight. Reasonably priced infra-red distance sensors can be accurate up to ±0.01mm, which would reduce the uncertainties to under 0.1%. However, in this case, other factors, such as surface inconsistencies of weights, or the slight rocking of hanging weights, and may limit such small uncertainty values.

Undeniably, however, systematic errors must be identified and resolved before other measures to reduce uncertainties would be meaningful.